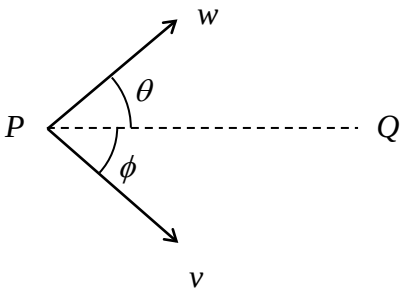
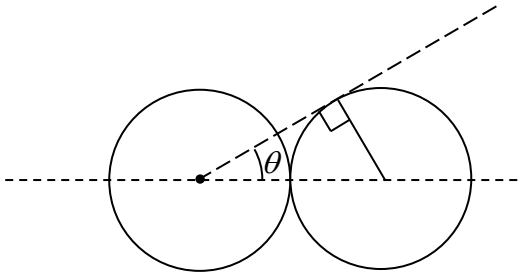
EDEXCEL MECHANICS M4 (6680) – JUNE 2004 **PROVISIONAL MARK SCHEME**

Question Number	Scheme	Marks
1. (*a*)	 $v_A = v_{AW} + v_W$ $\Rightarrow \begin{pmatrix} k \\ 0 \end{pmatrix} = \begin{pmatrix} v\cos\phi \\ -v\sin\phi \end{pmatrix} + \begin{pmatrix} w\cos\theta \\ w\sin\theta \end{pmatrix}$	M1
	$\Rightarrow v \sin \phi = w \sin \theta$*	A1 (2)
(*b*)	$k = v \sin \phi + w \sin \theta$	M1 A1
	$= \dfrac{v\sqrt{v^2 - w^2 \sin^2 \theta}}{v} + w \cos \theta$	M1
	$= \sqrt{v^2 - w^2 \sin^2 \theta} + w\cos\theta$	A1 (4)
		(6 marks)
2.	 $\sin \theta = \dfrac{a}{2a}$	M1
	$\Rightarrow \theta = 30°$	A1
	$v_1 + v_2 = u \cos \theta$	M1 A1
	$-v_1 + v_2 = eu \cos \theta$	M1 A1
	$\dfrac{u \sin \theta}{v_1} = \tan(\theta + 30°)$ (or equivalent)	M1 A1
	Producing an equation in *e* only	M1
	$e = \dfrac{1}{3}$	A1 c.s.o
		(11 marks)

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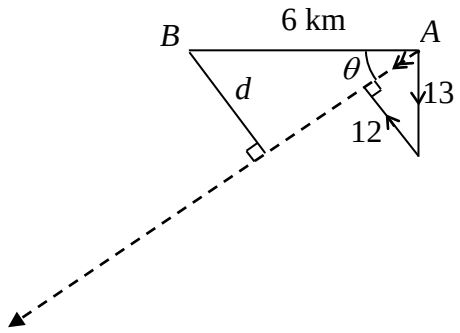
3. (*a*)	 Vector △ $\cos\theta = \frac{12}{13}$ (θ = 22.6°) Course is 360° – 22.6° = 337° (AWRT)	M1 A1 M1 A1 (4)
(*b*)	$v = \sqrt{13^2 - 12^2} = 5$ $t = \frac{6\cos\theta}{5} = 1.107$ Time is 1.06 p.m.	
(*c*)	$d = 6\sin\theta = 6 \times \frac{5}{13} = 2.31$ km (AWRT 2.3 km)	M1 A1 (2)
		(11 marks)

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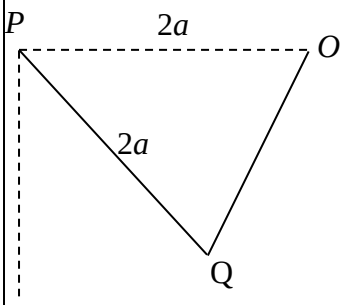
Question Number	Scheme	Marks
4. (*a*)	 $OQ = 4a \sin\theta$	B1
	$V = (-)\, mga \sin 2\theta + \frac{mg}{2\sqrt{3}2a}(4a\sin\theta - a)^2 + C$	B1; M1 A1
	$= -mga \sin 2\theta + \frac{mga^2}{4a\sqrt{3}}(16\sin^2\theta - 8\sin\theta + 1) + C$	M1
	$= -mga \sin 2\theta + \frac{mga}{4\sqrt{3}}(8(1-\cos 2\theta) - 8\sin\theta) + C$	M1
	i.e. $V = -\frac{mga}{\sqrt{3}}(2\cos 2\theta + \sqrt{3}\sin 2\theta + 2\sin\theta) + C$ *	A1 c.s.o (7)
(*b*)	$V'(\theta) = -\frac{mga}{\sqrt{3}}(-4\sin 2\theta + 2\sqrt{3}\cos 2\theta + 2\cos\theta)$	M1 A1
	$V'\left(\frac{\pi}{6}\right) = -\frac{mga}{\sqrt{3}}\left(-2\sqrt{3} + 2\sqrt{3}\frac{1}{2} + 2\frac{\sqrt{3}}{2}\right) = 0$	M1 A1 (4)
(*c*)	$V''(\theta) = \frac{mga}{\sqrt{3}}(+8\cos 2\theta + 4\sqrt{3}\sin 2\theta + 2\sin\theta)$	M1 A1
	Hence, $V''\left(\frac{\pi}{6}\right) = \frac{11mga}{\sqrt{3}} > 0 \therefore$ stable	M1 A1 c.s.o (4)
		(15 marks)

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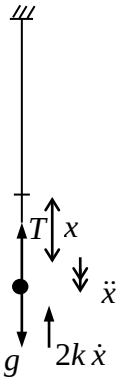
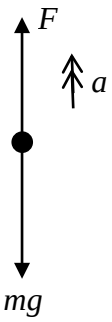
Question Number	Scheme	Marks
5. (*a*)	 R(↓), $mg - 2k\dot{x}m - T = m\ddot{x}$	
	$g - 2k\dot{x} - \frac{2ak^2x}{a} = \ddot{x}$	M1
	$\Rightarrow \ddot{x} + 2k\dot{x} + 2k^2x = g$ *	A1 c.s.o (4)
(*b*)	$t = 0, x = a$: $a = D + \frac{g}{2k^2} \Rightarrow D = a - \frac{g}{2k^2}$	M1 A1
	$\dot{x} = -ke^{-kt}(C\sin kt + D\cos kt) + -ke^{-kt}(C\cos kt - D\sin kt)$	M1 A1
	$t = 0, \dot{x} = 0$: $0 = -kD + kC \Rightarrow C = D$	
	$\Rightarrow C = a - \frac{g}{2k^2}$	A1 (5)
(*c*)	$\dot{x} = 0 \Rightarrow$ $C(\sin kt + \cos kt) + C(\cos kt - \sin kt)$	M1
	$\Rightarrow \sin kt = 0$	A1 ft
	$\Rightarrow kt = \pi$	
	$\Rightarrow t = \frac{\pi}{k}$	A1 (3)
(*d*)	When $t = \frac{\pi}{k}$, $x = -De^{-\pi} + \frac{g}{2k^2}$	M1
	$\frac{g}{2k^2} - e - \pi(a - \frac{g}{2k^2})$	A1 ft
	$\Rightarrow xe^{\pi} = \frac{g}{2k^2}(e^{\pi} + 1) - a$	
	> 0 (given)	M1
	$\Rightarrow g(e^{\pi} + 1 > 2k^2a$ *	A1 c.s.o (4)
		(16 marks)

Question Number	Scheme	Marks
6. (*a*)	 $F = \frac{kmg}{v}$	B1
	R(↑), $F - mg = ma$	M1
	$\frac{kmg}{v} - mg = mv\frac{dv}{dx}$	M1
	$g(k - v) = v^2\frac{dv}{dx}$ *	A1 (4)
(*b*)	$g\,dx = \frac{v^2}{k-v}dv$	M1
	$\int g\,dx = \int -v - k + \frac{k^2}{k-v}dv$	M1 A1
	$gx = -\frac{v^2}{2} - kv - k^2\ln(k - v) + (c)$	M1 A1
	$x = 0, v = 0$	
	$0 = 0 - 0 - k^2 \ln k + c$	M1
	$c = k^2 \ln k$	
	$gx = -\frac{v^2}{2} - kv - k^2 \ln\left(\frac{k}{k-v}\right)$ *	A1 (7)
(*c*)	Work done by engine = Energy gain	
	$kmgt = \frac{1}{2}mv^2 + mgx$	M1 A1
	$kmgt = mk^2 \ln\left(\frac{k}{k-v}\right) - mkv$	M1
	$\Rightarrow t = \frac{k}{g}\ln\left(\frac{k}{k-v}\right) - \frac{v}{g}$	A1 (5)
		(16 marks)